



**ENGINEERING
ADMISSIONS ASSESSMENT**

D564/11

Wednesday 30 October 2019

60 minutes

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

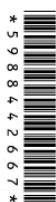
You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 33 printed pages and 3 blank pages.



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PART A Mathematics and Physics

1 Evaluate

A 0

B $2\sqrt{7}$

C $4\sqrt{7}$

D $2\sqrt{21}$

E 10

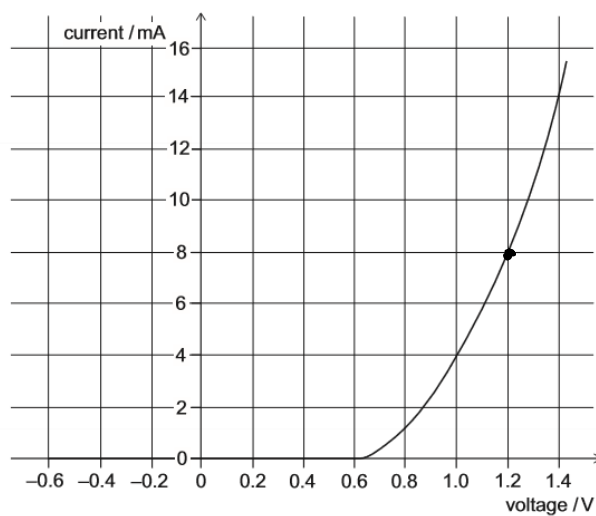
☒ F $4\sqrt{21}$

G 20

$$(\sqrt{7} + \sqrt{3})^2 - (\sqrt{7} - \sqrt{3})^2$$

$$\begin{aligned} a^2 - b^2 &= (a+b)(a-b) \\ &= (\sqrt{7} + \sqrt{3} + \sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}) \\ &= (2\sqrt{7})(2\sqrt{3}) \\ &= \underline{\underline{4\sqrt{21}}} \end{aligned}$$

- 2 The current–voltage graph for a diode is shown.



The diode is connected in series with a resistor and a 6.0 V battery. The current in the circuit is 8.0 mA.

What is the resistance of the resistor?

(Assume that the battery has negligible resistance.)

- A 0.15 Ω
- B 0.60 Ω
- C 0.75 Ω
- D 4.8 Ω
- E 150 Ω
- ☒ F 600 Ω
- G 750 Ω

From Graph, $V_{\text{diode}} = \underline{\underline{1.2 \text{ V}}}$

$V_{\text{resistor}} = 6 - 1.2$
 $= 4.8 \text{ V}$

$R = \frac{V}{I} = \frac{4.8}{0.008} = \frac{4800}{8} = \underline{\underline{600 \Omega}}$

- 3 The equation gives y in terms of x :

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

Which one of the following is a rearrangement for x in terms of y ?

A $x = -2 \pm 2\sqrt{\frac{3-y}{4}}$

B $x = -2 \pm 2\sqrt{\frac{4-y}{3}}$

C $x = 1 \pm \sqrt{\frac{3-y}{4}}$

D $x = 1 \pm 2\sqrt{\frac{3-y}{4}}$

☒ E $x = 2 \pm 2\sqrt{\frac{3-y}{4}}$

F $x = 2 \pm 2\sqrt{\frac{4-y}{3}}$

G $x = 2 \pm 2\sqrt{\frac{3+y}{4}}$

$$y - 3 = -4\left(1 - \frac{x}{2}\right)^2$$

$$\frac{3-y}{4} = \left(1 - \frac{x}{2}\right)^2$$

$$1 - \frac{x}{2} = \pm \sqrt{\frac{3-y}{4}}$$

$$\frac{x}{2} = 1 \pm \sqrt{\frac{3-y}{4}}$$

$$\therefore x = 2 \pm 2\sqrt{\frac{3-y}{4}} //$$

- 4 Two electromagnetic waves P and Q travel in a vacuum and the ratio of their wavelengths is:

$$\frac{\text{wavelength of P}}{\text{wavelength of Q}} = 1.0 \times 10^8$$

$v = f\lambda = \text{constant}$
 $\therefore \lambda_P \lambda_Q = f_Q \lambda_Q$

Which row in the table shows the ratio of their speeds, the ratio of their frequencies, and identifies the possible natures of P and Q?

$\frac{\lambda_P}{\lambda_Q} = \frac{f_Q}{f_P}$
 \downarrow
 $= 10^8$

	$\frac{\text{speed of P}}{\text{speed of Q}}$	$\frac{\text{frequency of P}}{\text{frequency of Q}}$	nature of P	nature of Q
<input checked="" type="radio"/> A	1.0	1.0×10^{-8}	microwave	X-ray
B	1.0	1.0×10^{-8}	microwave	radio wave
<input checked="" type="radio"/> C	1.0	1.0×10^8	infrared	ultraviolet
<input checked="" type="radio"/> D	1.0	1.0×10^8	visible light	infrared
<input checked="" type="radio"/> E	1.0×10^8	1.0	gamma	X-ray
<input checked="" type="radio"/> F	1.0×10^8	1.0	gamma	infrared
<input checked="" type="radio"/> G	1.0×10^8	1.0×10^{16}	infrared	radio wave
<input checked="" type="radio"/> H	1.0×10^8	1.0×10^{16}	visible light	ultraviolet

- 5 The resistance to the motion of a car is directly proportional to the square of the speed of the car.

The car increases its speed by 20%.

What is the percentage increase in the resistance to the motion of the car?

- A 20%
 B 24%
☒ C 44%
 D 120%
 E 224%
 F 240%
 G 400%

$R \propto v^2$
 $R = kv^2$
 $R_0 = kv_0^2$
 $R_1 = k(1.2v_0)^2 = 1.44kv_0^2$
 $\frac{R_1}{R_0} = \frac{1.44kv_0^2}{kv_0^2} = 1.44$

- 6 A water-tight cylinder with a thin, freely moving piston contains $2.0 \times 10^{-3} \text{ m}^3$ of trapped air at atmospheric pressure of $1.0 \times 10^5 \text{ Pa}$.

When the cylinder is submerged in water of constant density 1000 kg m^{-3} , the volume of air in the cylinder decreases to $4.0 \times 10^{-4} \text{ m}^3$.

The piston is at a depth h below the surface of the water and the water surface is open to the atmosphere.

What is the depth h ?

(gravitational field strength = 10 N kg^{-1} ; assume that the temperature of the air remains constant and that air is an ideal gas)

☒ A 40 m

B 50 m

C 60 m

D 400 m

E 500 m

F 600 m

$$pV = \text{Constant}$$

$$1 \times 10^5 \times 2 \times 10^{-3} = p_2 \times 4 \times 10^{-4}$$

$$p_2 = \frac{200}{4 \times 10^{-4}} = 0.5 \times 10^6 = 5 \times 10^5$$

$$\Delta p = \rho g \Delta h$$

$$5 \times 10^5 - 1 \times 10^5 = 4 \times 10^5 = 1000 \times 10 \times \Delta h$$

$$\Delta h = \frac{400000}{10000} = 40 \text{ m}$$

- 7 The equation of a curve is $y = px^2 + qx$ where p and q are constants.

The curve passes through the points $(2, 6)$ and $(4, -4)$.

What is the value of $q - p$?

A 1

B 2

C 5

D 6

☒ E 9

F 16

$$6 = p(2^2) + q(2) = 4p + 2q$$

$$\therefore 2p + q = 3$$

$$-4 = p(4^2) + q(4)$$

$$\therefore 16p + 4q = -4 \Rightarrow 4p + q = -1$$

$$-2p + q = 3$$

$$2p = -4 \therefore p = -2$$

$$2(-2) + q = 3 \therefore q = 3 - -4 = 7$$

$$q - p = 7 - -2 = 9$$

- 8 The secondary coil of an ideal, 100% efficient transformer is connected to a resistor by cables of total resistance 1500Ω . The current in the primary coil is 4.0A . There are 240 turns in the primary coil and 4800 turns in the secondary coil.

What is the power produced as heat in the cables?

- ☒ A 60 W
 B 300 W
 C 6000 W
 D 24000 W
 E 120000 W
 F 9600000 W

$$\begin{aligned} \frac{V_p}{V_s} &= \frac{N_p}{N_s} & V_p I_p &= V_s I_s \\ \therefore \frac{I_s}{I_p} &= \frac{N_p}{N_s} \Rightarrow I_s = \frac{240}{4800} \times 4 \\ &= \frac{24}{48} \times \frac{1}{10} \times 4 \\ &= 0.2\text{A} \\ P &= I^2 R = 0.2^2 \times 1500 = 1500 \times 0.04 \\ &= 15 \times 4 = 60\text{W} \end{aligned}$$

- 9 Which of the following is a simplification of

$$4 - \frac{x(3x+1)}{x^2(3x^2-2x-1)}$$

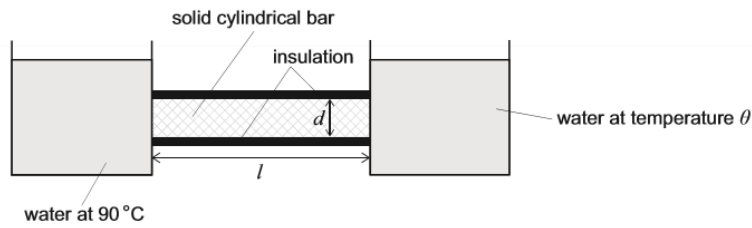
- A $\frac{12x^3-8x^2-7x-1}{x(3x-1)(x-1)}$
 B $\frac{4x^2+4x-1}{x(x+1)}$
 C $\frac{4x^2+4x+1}{x(x+1)}$
☒ D $\frac{4x^2-4x-1}{x(x-1)}$
 E $\frac{4x^2-4x+1}{x(x-1)}$
 F $\frac{12x^3-8x^2-x+1}{x(3x-1)(x-1)}$

$$\begin{aligned} &3x^2-2x-1 \\ &= 3x^2-3x+x-1 \\ &= 3x(x-1)+1(x-1) \\ &= (3x+1)(x-1) \\ \Rightarrow 4 - \frac{x(3x+1)}{x(3x+1)(x-1)} \\ &= 4 - \frac{1}{x(x-1)} \\ &= \frac{4x(x-1)-1}{x(x-1)} = \frac{4x^2-4x-1}{x(x-1)} // \end{aligned}$$

- 10 Two tanks of water are connected by a solid cylindrical copper bar of length l and diameter d .

The bar is insulated.

One tank contains water at 90°C and the other tank contains water at temperature θ .



For which of the following conditions is thermal energy conducted along the bar at the lowest rate?

	l/m	d/cm	$\theta/^\circ\text{C}$
A	0.40	4.0	20
B	0.40	4.0	40
C	0.40	8.0	20
D	0.40	8.0	40
E	0.80	4.0	20
F	0.80	4.0	40
G	0.80	8.0	20
H	0.80	8.0	40

Lowest rate:

Greatest l

Lowest d

Lowest $\Delta\theta$

$\rightarrow \Delta\theta = 70$

$\rightarrow \Delta\theta = 90 - 40 = 50$

- 11 The ball for a garden game is a solid sphere of volume 192 cm^3 .

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3 , of the children's ball?

A 48

☒ B 81

C 96

D 108

E 144

$$\frac{4}{3}\pi r^3 = 192$$

$$\frac{4}{3}\pi \left(\frac{3}{4}r\right)^3 = 192 \times \frac{3^3}{4^3}$$

$$= \frac{192}{64} \times 27 = 3 \times 27 = 81$$

- 12 The radioactive isotope X becomes the stable isotope Y after a succession of decays involving only the emission of alpha and beta (β^-) particles.

During the decay of one nucleus from X to Y, a total of seven particles are emitted. It is known that more of these particles are alpha particles than beta particles.

The atomic number of X is Z and the mass number of X is A.

Which row in the table could give the atomic number and the mass number of Y?

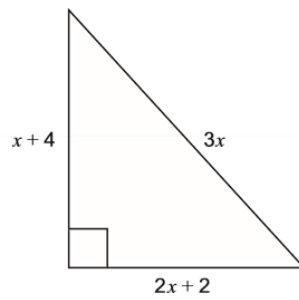
	atomic number of Y	mass number of Y
<input checked="" type="radio"/> A	$Z - 2$	$A - 12$
<input checked="" type="radio"/> B	$Z - 5$	$A - 8$
<input checked="" type="radio"/> C	$Z - 8$	$A - 20$
D	$Z - 10$	$A - 24$
E	$Z - 11$	$A - 16$

$\alpha \rightarrow A \downarrow \text{ by } 4$
 $Z \downarrow \text{ by } 2$
 $\beta^- \rightarrow A \text{ stays the same}$
 $Z \uparrow \text{ by } 1$

$N_\alpha > N_{\beta^-} \therefore$ There must be at least 4 α particles.

$$C \rightarrow 5 \times \alpha, 2 \times \beta^- \rightarrow Z = Z_0 - 10 + 2 = \underline{\underline{Z_0 - 8}}$$

- 13 The diagram shows a right-angled triangle, with sides of length $x + 4$, $2x + 2$ and $3x$, all in cm.



[diagram not to scale]

What is the area, in cm^2 , of the triangle?

- A 10
- B 12
- C 28
- D 36
- E 40
- ☒ F 54
- G 70

Area
 $= \frac{1}{2}(x+4)(2x+2)$
 $= \frac{1}{2}(9)(12)$
 $= \underline{\underline{54}}$

$(x+4)^2 + (2x+2)^2 = (3x)^2$
 $x^2 + 8x + 16 + 4x^2 + 8x + 4 = 9x^2$
 $5x^2 + 16x + 20 = 9x^2$
 $\therefore 4x^2 - 16x - 20 = 0 \Rightarrow x^2 - 4x - 5 = 0$
 $(x-5)(x+1) = 0$
 length $> 0 \Rightarrow x = 5$

- 14 The kinetic energy of an object of mass 4.0 kg , travelling in a straight line, increases from 32 J to 200 J in 3.0 seconds due to a constant resultant force.

What is the value of this resultant force?

- A 2.0 N
- B 4.0 N
- ☒ C 8.0 N
- D 24 N
- E 28 N
- F 56 N

$32 = \frac{1}{2}mv^2 = 2v^2$
 $v = \sqrt{\frac{32}{2}} = 4 \text{ m/s}$
 $200 = \frac{1}{2}mv_1^2 = 2v_1^2$
 $v_1 = \sqrt{\frac{200}{2}} = 10 \text{ m/s}$
 $a = \frac{v_1 - v_0}{t} = \frac{10 - 4}{3} = 2 \text{ ms}^{-2}$
 $F = ma = \underline{\underline{8 \text{ N}}}$

- 15 PR and QS are the diagonals of a rhombus $PQRS$.

$$PR = (3x + 2) \text{ cm}$$

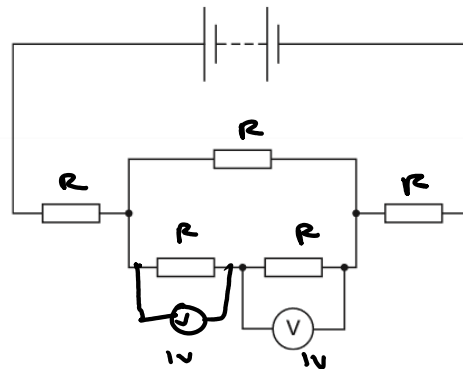
$$QS = (8 - 2x) \text{ cm}$$

The area of $PQRS$ is 11 cm^2 .

What is the difference, in cm, between the two possible lengths of PR ?

- A $2\frac{2}{3}$
 B $4\frac{1}{2}$
 C $5\frac{1}{3}$
 (D) 8
 E 14
- Area of rhombus = $\frac{\text{Product of Diagonals}}{2}$
- $$11 = \frac{1}{2}(3x+2)(8-2x)$$
- $$22 = -6x^2 + 20x + 16$$
- $$6x^2 - 20x + 6 = 0$$
- $$6x^2 - 18x - 2x + 6 = 0$$
- $$6x(x-3) - 2(x-3) = 0 \therefore (6x-2)(x-3) = 0$$
- $$x = 3 \rightarrow PR = 3(3) + 2 = 11$$
- $$x = \frac{1}{3} \rightarrow PR = 3\left(\frac{1}{3}\right) + 2 = 3$$
- $$\underline{11 - 3 = 8}$$

- 16 In the following circuit, all five resistors have the same resistance.



The reading on the voltmeter is 1.0 V.

What is the voltage across the battery?

- A 4.0 V
- B 5.0 V
- C 6.0 V
- D 7.0 V
- ☒ E 8.0 V
- F 9.0 V
- G 10 V

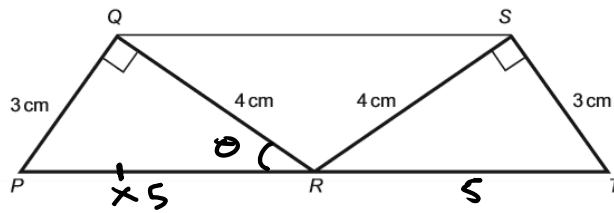
Let resistance of 1 resistor = R

Parallel resistance = $\frac{R \times 2R}{R + 2R} = \frac{2R^2}{3R} = \frac{2R}{3}$

V across the parallel resistors = $1V + 1V = 2V$

$\frac{2R/3}{2R + 2R/3} \times V = 2 \therefore \frac{1}{4} V = 2$
 $V = 8V //$

17



[diagram not to scale]

The diagram shows two congruent right-angled triangles PQR and TSR with right angles at Q and S , respectively.

$$PQ = TS = 3 \text{ cm}$$

$$QR = SR = 4 \text{ cm}$$

PRT is a straight line.

What is the length, in cm, of QS ?

- A 4
- B $3\sqrt{2}$
- C 5.2
- D $4\sqrt{2}$
- ☒ E 6.4
- F 8.2
- G 10

$$PR = RT = 5 \text{ cm (Pythagoras)}$$

$$\sin \theta = \frac{3}{5}$$

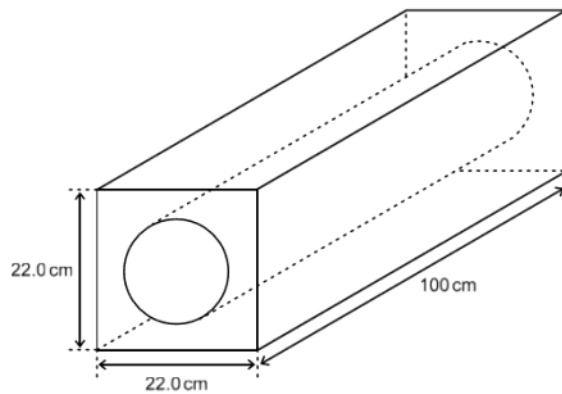
$$XR = 4 \cos \theta$$

$$\sin \theta = \frac{3}{5} \angle \cos \theta = \frac{4}{5}$$

$$XR = 4 \times \frac{4}{5} = \frac{16}{5}$$

$$QS = 2XR = \frac{32}{5} = 6.4$$

- 18 A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:



[diagram not to scale]

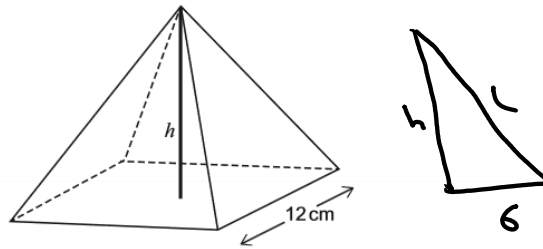
The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is 0.100 g cm^{-3} .

What is the mass of the block?

(take π to be $\frac{22}{7}$)

- A 1.32 kg
- B 3.30 kg**
- C 13.2 kg
- D 33.0 kg
- E 132 kg
- F 330 kg
- G 1320 kg
- H 3300 kg

$$\begin{aligned}
 \text{Volume of block} &= 22^2(100) \\
 \text{Volume of hole} &= \pi r^2(100) \\
 V_{\text{material}} &= 22^2(100) - \frac{22}{7}(7^2)(100) \\
 &= 484(100) - 154(100) \\
 &= 330(100) = 33000 \\
 \text{Mass} &= \rho \times V = 0.1 \times 33000 \\
 &= \underline{3300 \text{ g}}
 \end{aligned}$$



[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm^3 , is equal to the total surface area of the pyramid, in cm^2 .

What is the value of h ?

$$SA_{\text{tot}} = A_{\text{base}} + 4 \times A_{\text{triangle}}$$

(volume of pyramid = $\frac{1}{3} \times \text{area of base} \times \text{vertical height}$)

A $\frac{72}{35}$

B $2\sqrt{3}$

C 6

D $\frac{144}{23}$

(E) 8

F $2\sqrt{21}$

$$\frac{1}{3} \times 12^2 \times h = 12^2 + 4 \left(\frac{1}{2} \times 12 \times \sqrt{h^2 + 6^2} \right)$$

$$48h = 144 + 24\sqrt{h^2 + 36}$$

$$2h - 6 = \sqrt{h^2 + 36} \quad \therefore 4h^2 - 24h + 36 = h^2 + 36$$

$$3h^2 - 24h = 0$$

$$h(3h - 8) = 0$$

$$\underline{h = 8}$$

- 20 A sample initially contains equal numbers of atoms of a radioactive isotope X and a stable isotope Y.

Isotope X has a half-life of 3 years and decays in a single stage to the stable isotope Y.

What is the ratio

number of atoms of X : number of atoms of Y

in the sample 6 years later?

A The sample contains only isotope Y.

☒ B 1:7

C 1:4

D 1:3

E 7:4

$$\begin{array}{ccc} T=0 & \frac{X}{1} & \frac{Y}{1} \\ T=3 & \frac{1}{2} & \frac{3}{2} \\ T=6 & \frac{1}{4} & \frac{7}{4} \end{array}$$

$$X:Y = \frac{1}{4} : \frac{7}{4} = \underline{\underline{1:7}}$$

PART B Advanced Mathematics and Advanced Physics

- 21 Find the area of the shape bounded by the four lines:

$$2y + x = 4$$

$$x = -6$$

$$x = 0$$

$$y = 0$$

A 4

B 12

☒ C 21

D 25

E 27

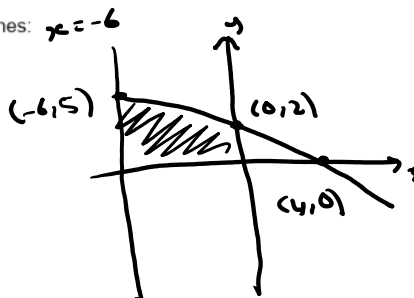
F 30

$$2y + x = 4$$

$$y = 2 - \frac{x}{2}$$

$$Area = \frac{1}{2}(a+b) \cdot h$$

$$= \frac{1}{2}(5+2) \cdot 6 = 3 \times 7 = 21$$



- 22 A hydroelectric power station uses the water in a reservoir to power the generators. The water falls through a vertical height of 150 m to the turbines which power the generators.

The efficiency of the power station is 90% and the output power of the power station is 1800 MW.

The gravitational field strength is 10 N kg^{-1} and the density of water is 1000 kg m^{-3} .

What volume of water passes through the turbines in one minute?

A $6.48 \times 10^4 \text{ m}^3$

B $7.20 \times 10^4 \text{ m}^3$

☒ C $8.00 \times 10^4 \text{ m}^3$

D $6.48 \times 10^7 \text{ m}^3$

E $7.20 \times 10^7 \text{ m}^3$

F $8.00 \times 10^7 \text{ m}^3$

$$\text{Power input} = \frac{1800 \text{ MW}}{0.9}$$

$$= 2000 \text{ MW}$$

$$\text{CPE} = mgh = \rho Vgh$$

$$\rho Vgh = 2000 \times 10^6 \times 60$$

$$V \times 1000 \times 150 = 2000 \times 10^6 \times 60$$

$$V = \frac{1200000}{15} = 80000$$

23 The curve

$$y = x^3 + px^2 + qx + r$$

has a local maximum when $x = -1$ and a local minimum when $x = 3$

What is the value of p ?

A -9

B -3

C -1

D 1

E 3

F 9

$$\frac{dy}{dx} = 3x^2 + 2px + q = 0$$

$$3(-1)^2 + 2p(-1) + q = 0$$

$$2p - q = 3 \quad (1)$$

$$3(3^2) + 2p(3) + q = 0$$

$$6p + q = -27 \quad (2)$$

$$(1) + (2) \rightarrow 8p = -24 \therefore p = -3$$

- 24 A car P of mass 1000 kg is travelling north at 30 m s^{-1} along a straight, horizontal road when it hits another car Q which is directly ahead of P and travelling in the same direction. Car Q has a mass of 500 kg and is travelling at 20 m s^{-1} .

The collision lasts for 0.20 s and immediately after the collision car Q is moving north at 30 m s^{-1} .

What is the speed of P immediately after the collision and what is the size of the average resultant force that acts on Q during the collision?

(Assume that no external forces act on the cars during the collision.)

	speed of P / m s^{-1}	average force on Q / N
A	20	25 000
B	20	50 000
C	20	100 000
D	20	125 000
E	25	25 000
F	25	50 000
G	25	100 000
H	25	125 000

B: $\frac{30}{\text{P}}$ $\frac{20}{\text{Q}}$
 $\frac{10}{\text{P}}$ $\frac{30}{\text{Q}}$

$$30(1000) + 20(500) = 1000x + 30(500)$$

$$3000 + 1000 = 10x + 1500$$

$$10x = 2500 \therefore x = 25$$

$$F_Q = \frac{\Delta p}{\Delta t} = \frac{500(30-20)}{0.2}$$

$$= 5000 \times 5$$

$$= 25000 \text{ N}$$

- 25 When simplified, $\frac{1}{(1-\sqrt{2})^3}$ is written in the form $a+b\sqrt{2}$ where a and b are integers.

What is the value of b ?

- A -7
 B -5
 C -1
 D 1
 E 5
 F 7

$$\begin{aligned} (1-\sqrt{2})^3 &= 1 - 3\sqrt{2} + 6 - 2\sqrt{2} \\ &= 7 - 5\sqrt{2} \\ \frac{1}{7-5\sqrt{2}} &\times \frac{7+5\sqrt{2}}{7+5\sqrt{2}} \\ &= \frac{7+5\sqrt{2}}{49-50} = -7-5\sqrt{2} \end{aligned}$$

- 26 A metal wire of length 0.50 m has a uniform cross-sectional area of $4.0 \times 10^{-7} \text{ m}^2$.

There is a current of 4.0 A in the wire.

What is the potential difference across the ends of the wire?

(resistivity of the metal = $1.6 \times 10^{-7} \Omega \text{ m}$)

- A 0.05 V
 B 0.20 V
 C 0.80 V
 D 3.2 V
 E 5.0 V
 F 20 V

$$\begin{aligned} R &= \frac{\rho L}{A} \\ V &= \frac{I \rho L}{A} = \frac{4 \times 1.6 \times 10^{-7} \times 0.5}{4 \times 10^{-7}} \\ &= 0.8 \text{ V} \end{aligned}$$

27 It is given that

$$7\cos x + \tan x \sin x = 5$$

where $0^\circ < x < 90^\circ$

What are the possible values of $\tan x$?

A $\frac{1}{2}$ or $\frac{1}{3}$

B $\frac{1}{\sqrt{3}}$ or $\frac{1}{2\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$ or $\frac{2\sqrt{2}}{3}$

☒ D $\sqrt{3}$ or $2\sqrt{2}$

E 3 or 2

$$7\cos x + \frac{\sin^2 x}{\cos x} = 5$$

$$7 + \tan^2 x = 5\sec x$$

$$7 + \sec^2 x - 1 = 5\sec x$$

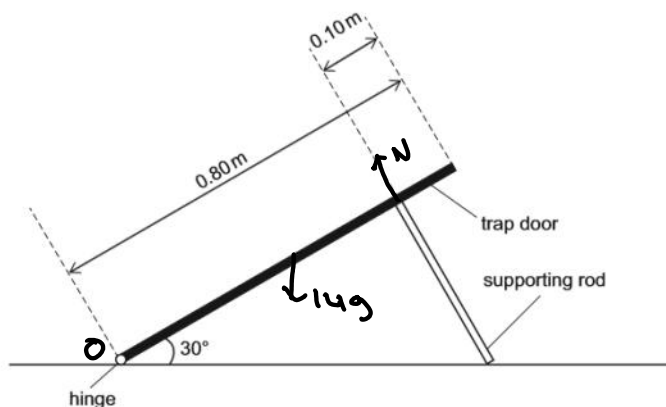
$$\sec^2 x - 5\sec x + 6 = 0$$

$$\sec x = 2 \text{ or } \sec x = 3$$

$$\sec^2 x = 4 \text{ or } \sec^2 x = 9$$

$$1 + \tan^2 x = 4 \rightarrow \tan x = \sqrt{3} \quad 1 + \tan^2 x = 9 \rightarrow \tan x = 2\sqrt{2}$$

- 28 A uniform square trap door of side 0.80 m and mass 14 kg has a smooth hinge at one edge and is held open at an angle of 30° to the horizontal. It is supported by a single rigid rod placed so that it meets the surface of the trap door at 90° at a distance 0.10 m from the top edge of the trap door, as shown.



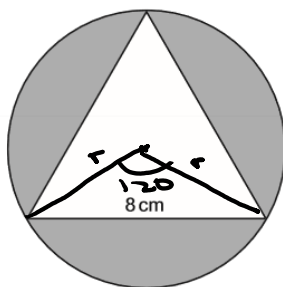
What is the normal contact force exerted on the trap door by the rod?

(gravitational field strength = 10 N kg^{-1})

- A 40 N
- B $35\sqrt{3} \text{ N}$
- C $40\sqrt{3} \text{ N}$**
- D 80 N
- E $80\sqrt{3} \text{ N}$
- F $280\frac{\sqrt{3}}{3} \text{ N}$

$$\begin{aligned} \text{Co: } 14g(0.4\cos 30) &= N(0.7) \\ 14 \times 10 \times 0.4 \times \frac{\sqrt{3}}{2} &\approx 0.7N \\ \frac{28 \times 86.53}{7} &\approx 0.7N \therefore N = \underline{\underline{40\sqrt{3}}} \end{aligned}$$

- 29 An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.



What is the total of the three areas shaded in the diagram, in cm^2 ?

A $8(2\pi - 3)$

B $24(\pi - \sqrt{3})$

C $48(4\pi - \sqrt{3})$

D $\frac{16}{3}(4\pi - 6 - 3\sqrt{3})$

☒ E $\frac{16}{3}(4\pi - 3\sqrt{3})$

Area of triangle = $\frac{1}{2}AB \sin C$

$= \frac{1}{2} \times 64 \times \sin 60$

$= \frac{32\sqrt{3}}{2} = 16\sqrt{3}$

cosine Rule

$8^2 = r^2 + r^2 - 2r(r) \cos 120$

$64 = 2r^2 - 2r^2(-\frac{1}{2}) = 3r^2$

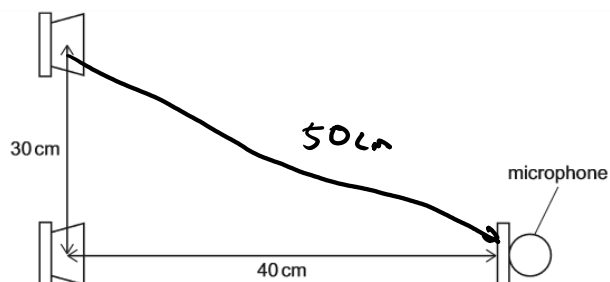
$\therefore r = \frac{8}{\sqrt{3}} \therefore \text{Circle area} = \pi r^2 = \frac{64\pi}{3}$

Shaded = $\frac{64\pi}{3} - 16\sqrt{3}$

- 30 Two small loudspeakers are placed side by side 30 cm apart.

They are connected to the same signal generator so that they emit sound of frequency 400 Hz in phase with one another.

The sounds both reach a microphone placed 40 cm directly in front of one of the two loudspeakers as shown.



What is the phase difference between waves from the loudspeakers as they arrive at the microphone?

(speed of sound = 320 m s^{-1})

A 30°

B 36°

☒ C 45°

D 60°

E 72°

F 90°

G 120°

$$\Delta\phi = \frac{360}{\lambda} \cdot \Delta x$$
$$\Delta x = \text{path diff.} = 50 \text{ cm} - 40 \text{ cm} = \underline{\underline{10 \text{ cm}}}$$
$$\Delta\phi = \frac{360 \times 0.1}{0.8} = 45^\circ$$
$$= 36 \times \frac{10}{8} = \frac{360}{8} = \underline{\underline{45^\circ}}$$

- 31 Which one of the following is the real solution of the equation

$$3 \times 5^{2x+1} - 5^x - 2 = 0$$

A $x = \log_5\left(\frac{1}{3}\right)$

B $x = \log_5\left(\frac{2}{5}\right)$

C $x = \log_5\left(\frac{3}{5}\right)$

D $x = \log_5\left(\frac{2}{3}\right)$

E $x = \log_5\left(\frac{5}{3}\right)$

F $x = \log_5\left(\frac{5}{2}\right)$

$$\begin{aligned} 5^{2x+1} &= 5 \times 5^{2x} \\ 3 \times 5 \times 5^{2x} - 5^x - 2 &= 0 \\ \text{Let } v &= 5^x \\ 15v^2 - v - 2 &= 0 \\ 15v^2 + 5v - 6v - 2 &= 0 \\ 5v(3v + 1) - 2(3v + 1) &= 0 \\ v - 5^x &= -\frac{1}{3} \text{ or } \frac{2}{3} \end{aligned}$$

- 32 An astronaut on the Moon throws a ball vertically upwards. The ball has a mass of 2.0 g and is thrown upwards at 80 m s^{-1} .

What is the maximum height gained by the ball?

(gravitational field strength close to the Moon's surface = 1.6 N kg^{-1})

A 25 m

B 50 m

C 320 m

D 2000 m

E 3200 m

F 4000 m

$$\begin{aligned} \frac{1}{2}mv^2 &= mgh \\ \frac{80^2}{2} &= 1.6h \\ h &= \frac{80 \times 80}{2 \times 1.6} \\ &= 40 \times 50 = \underline{\underline{2000 \text{ m}}} \end{aligned}$$

- 33 For a particular function $f(x)$, it is given that:

$$\int_{-2}^2 2f(x)dx + \int_2^4 f(x)dx = 4$$

$2x \quad \rightarrow$

and also:

$$\int_{-2}^2 5f(x)dx - \int_{-2}^4 f(x)dx = 7$$

$5x \quad 2$

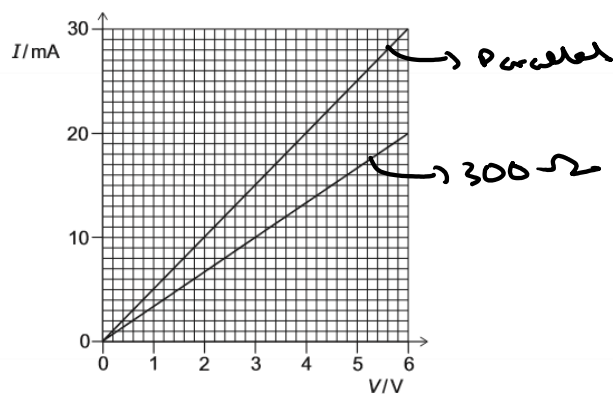
Find the value of $\int_2^4 f(x)dx$

- ☒ A $\frac{1}{3}$
 B $\frac{11}{7}$
 C $\frac{11}{6}$
 D $\frac{13}{6}$
 E $\frac{13}{3}$

$$\begin{aligned} 2x + y &= 4 \\ 5x - z &= 7 \\ z &= x + y \\ \therefore 4x - y &= 7 \\ + \quad 2x - y &= 4 \\ \hline 6x &= 11 \therefore x = \frac{11}{6} \\ y &= 4 - \frac{2 \times 11}{6} = \frac{1}{3} \end{aligned}$$

- 34 A student has one $300\ \Omega$ resistor and another resistor of resistance R .

The student plots a graph of current I against potential difference V for the $300\ \Omega$ resistor and then for both resistors connected in parallel.



What is the resistance R ?

- A $3.3\ \Omega$
- B $5.0\ \Omega$
- C $10\ \Omega$
- D $100\ \Omega$
- E $200\ \Omega$
- F $600\ \Omega$**
- G $1000\ \Omega$

$$R_{\text{tot}} = \frac{6}{0.03} = 200\ \Omega$$

$$\frac{300 + R}{300 + R} = 200$$

$$3R = 600 + 2R$$

$$R = \underline{600}$$

35 Given that

$$f(x) = \int_0^x (3+2t)^7 dt$$

what is the coefficient of x^4 in the expansion of $f(x)$ in powers of x ?

A 70

B 162

C $\frac{2835}{4}$

D 3024

☒ E 5670

F 15 120

G 22 680

$$\begin{aligned} (3+2t)^7 &= 3^7 + 7(3^6)(2t) + \dots + 7(3^2 \times 3^4 \times (2t)^3) \\ &= \frac{7!}{3! \times 4!} \times 3 \times 3 \times 3 \times 3 \times 8t^3 = \frac{7 \times 6 \times 5 \times 3^4 \times 8t^3}{3 \times 2 \times 1} \\ &= 35 \times 81 \times 8 \\ x^4 \text{ coefficient} &= \frac{35 \times 81 \times 8}{4} = 70 \times 81 = 5670 \end{aligned}$$

36 A light, vertical, copper wire of length 2.4 m and uniform cross-sectional area $2.0 \times 10^{-6} \text{ m}^2$ supports a load of mass 4.0 kg.

The Young modulus of copper is $1.2 \times 10^{11} \text{ Pa}$.

What is the strain energy in the wire?

(gravitational field strength = 10 N kg^{-1} ; assume that the wire obeys Hooke's law and that the cross-sectional area remains constant)

A $8.0 \times 10^{-5} \text{ J}$

B $1.7 \times 10^{-4} \text{ J}$

C $4.0 \times 10^{-4} \text{ J}$

☒ D $8.0 \times 10^{-3} \text{ J}$

E $4.0 \times 10^{-2} \text{ J}$

F $1.6 \times 10^{-2} \text{ J}$

$$\begin{aligned} \text{Strain energy} &= \frac{1}{2} Fx \\ E &= \frac{F}{A} = \frac{F}{A} \div \frac{x}{L} \therefore x = \frac{FL}{AE} \\ \text{Energy} &= \frac{F}{2} \times \frac{FL}{AE} = \frac{40^2 \times 2.4}{2 \times 1.2 \times 10^{11} \times 2 \times 10^{-6}} \\ &= \frac{800}{10^5} = 8 \times 10^{-3} \end{aligned}$$

- 37 The three internal angles in a triangle are α , β and θ , and

$$3\tan\alpha - 2\sin\beta = 2$$

$$5\tan\alpha + 6\sin\beta = 8$$

What is the value of θ in degrees?

- A 15
- B 45
- C 75
- ☒ D 105
- E 135

$$\begin{array}{r} 3x - 2y = 2 \quad \times 3 \\ 5x + 6y = 8 \quad \times 1 \\ \hline 9x - 6y = 6 \\ 5x + 6y = 8 \\ \hline 4x = 14 \quad \therefore x = \underline{\underline{1}} \end{array}$$

$$\tan\alpha = 1 \quad \therefore \alpha = 45^\circ$$

$$3 - 2y = 2 \quad \therefore y = \frac{1}{2}$$

$$\sin\beta = \frac{1}{2} \quad \therefore \beta = 30^\circ$$

$$180 - (45 + 30) = 105^\circ$$

- 38 Light travelling in a transparent liquid strikes the surface from below. The angle between the surface of the liquid and the direction of travel of the light is α .

The light then travels along the surface between the liquid and the air as shown in the diagram.



Now, light travelling in air strikes the surface from above so that the angle between the surface and the direction of travel of this light is also α .

After the light strikes the surface from above, the angle between the surface and the direction of travel of the refracted light is β .

Which expression gives β ?

(all angles are in degrees)

- ☒ A $\cos \beta = \cos^2 \alpha$
☐ B $\cos \beta = \cos \alpha \sin^{-1} \left(\frac{1}{\alpha} \right)$
☐ C $\sin \beta = \sin^2 \alpha$
☐ D $\beta = 90 - (90 - \alpha)^2$
☐ E $\beta = 0$
☐ F $\beta = \alpha$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \sin(90 - \alpha) = 1 \sin 90$$

$$\underline{\underline{n_1 \cos \alpha = 1}}$$

$$1 \sin(90 - \alpha) = n_1 \sin(90 - \beta)$$

$$\cos \alpha = n_1 \cos \beta$$

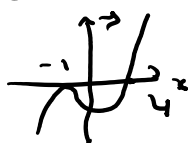
$$\cos \alpha = \frac{1}{\cos \alpha} \cdot \cos \beta$$

$$\therefore \cos \beta = \cos^2 \alpha$$

- 39 Find the complete set of values of x for which

$$x^3 - 2x^2 - 7x - 4 > 0$$

- ☐ A $x < -1$
☐ B $x > -1$
☐ C $-1 < x < 4$
☐ D $x < -1$ or $x > 4$
☐ E $x < 4$
☒ F $x > 4$



$$x^3 - 2x^2 - 7x - 4 = 0$$

$$f(1) = 1 - 2 - 7 - 4 \neq 0$$

$$f(-1) = -1 - 2 + 7 - 4 = 0$$

$$\Rightarrow (x+1)(x^2 + bx - 4)$$

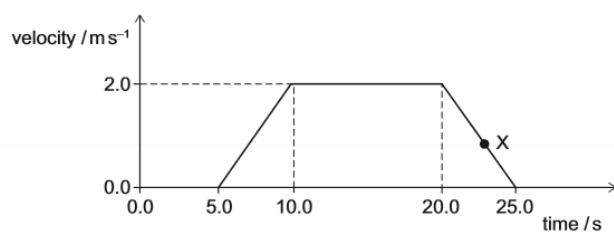
$$b - 4 = -7 \Rightarrow b = -3$$

$$\Rightarrow (x+1)(x-4)(x+1)$$

32

$$> 0 \text{ or } x > 4$$

- 40 The velocity–time graph is for an 80 kg person in a lift that is moving vertically upwards.



What is the magnitude of the contact force between the person and the lift floor at the time corresponding to X?

(gravitational field strength = 10 N kg^{-1})

- A 640 N
- ☒ B 768 N
- C 800 N
- D 832 N
- E 960 N



$$F - W = ma$$

$$F = 80(10) + \frac{80(0-2)}{5}$$

$$= 800 - 32$$

$$= \underline{\underline{768 \text{ N}}}$$

END OF TEST

Wednesday 30 October 2019

60 minutes

SECTION 2

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 2 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

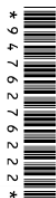
You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

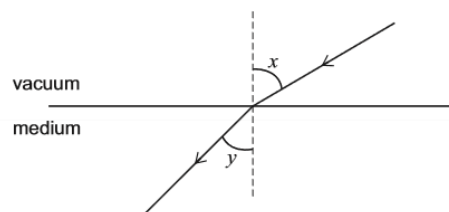
Please wait to be told you may begin before turning this page.

This question paper consists of 22 printed pages and 2 blank pages.



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- 1 The ray diagram shows light passing from a vacuum into a medium.



Two angles, x and y , are shown on the diagram.

When x is 60° , y is 45° .

When x is 45° , what is the value of $\sin y$?

- Ⓐ $\frac{1}{\sqrt{3}}$
 Ⓑ $\frac{2}{\sqrt{3}}$
 Ⓒ 1
 Ⓓ $\frac{\sqrt{3}}{2}$
 Ⓔ $\sqrt{3}$

$$1 \sin x = n_2 \sin y$$

$$\sin 60 = n_2 \sin 45$$

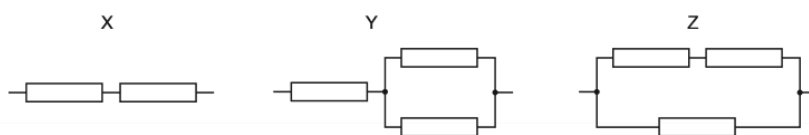
$$\frac{\sqrt{3}}{2} = n_2 \cdot \frac{\sqrt{2}}{2}$$

$$n_2 = \frac{\sqrt{3}}{\sqrt{2}}$$

$$1 \sin 45 = \frac{\sqrt{3}}{\sqrt{2}} \sin y$$

$$\sin y = \frac{\sqrt{2}}{2} \div \frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{\sqrt{3}}$$

- 2 Identical resistors are used to produce three different arrangements X, Y and Z.



Each arrangement is connected, in turn, across the same battery which has a negligible internal resistance.

The total power developed in each of the arrangements is determined.

What is the order of the arrangements when placed in order of increasing power?

A X, Y, Z

B X, Z, Y

C Y, X, Z

D Y, Z, X

E Z, X, Y

F Z, Y, X

$$R_x = 2R$$

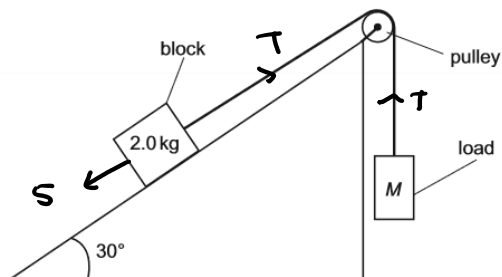
$$R_y = R + \frac{R \times R}{R + R} = \frac{3R}{2}$$

$$R_z = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$

$$P = \frac{V^2}{R} \therefore P \propto \frac{1}{R} \therefore \uparrow R \Rightarrow \downarrow P$$

- 3 A block of mass 2.0 kg is on a plane which is inclined to the horizontal at an angle of 30° .

The block is attached to a load of mass M by a light, inextensible string which passes over a smooth pulley.



The load moves downwards at a constant speed.

A constant friction force of 5.0 N acts on the block while it moves.

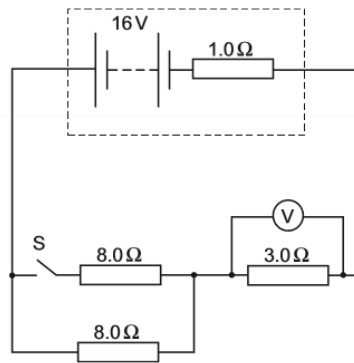
What is the value of M ?

(gravitational field strength = 10 N kg^{-1} ; assume that air resistance is negligible)

- A 0.50 kg
- B 1.0 kg
- ☒ C 1.5 kg
- D 2.5 kg
- E 4.0 kg
- F 6.0 kg

$$\begin{aligned} T &= 2g \sin 30 + S = 10 + 5 = 15 \\ T &= Mg \rightarrow \text{constant speed} \\ 15 &= 10M \therefore M = \underline{\underline{1.5\text{ kg}}} \end{aligned}$$

- 4 The battery in the circuit shown has an emf of 16 V and an internal resistance of 1.0 Ω .



Which line in the table gives the voltmeter readings when switch S is in its open and closed states?

	voltmeter reading / V	
	when S is open	when S is closed
A	$\frac{4.0}{3.0}$	2.0
B	4.0	6.0
C	4.0	2.4
D	6.0	2.4
E	6.0	4.0
F	$\frac{48}{11}$	$\frac{48}{19}$
G	$\frac{48}{11}$	$\frac{48}{7.0}$
H	$\frac{128}{11}$	$\frac{64}{7.0}$

Open:

$$R_{\text{tot}} = 1 + 8 + 3 = 12$$

$$V = \frac{3}{12} \times 16 = 4 \text{ V}$$

Closed:

$$R_{\text{tot}} = 1 + \frac{8 \times 8}{8 + 8} + 3 = 8$$

$$V = \frac{3}{8} \times 16 = 6 \text{ V}$$

- 5 A stationary wave is set up in a medium in which the speed of the wave is 3.2 m s^{-1} .

The stationary wave is formed by the superposition of two longitudinal waves, each of amplitude 1.5 cm , travelling in opposite directions.

The distance between adjacent nodes in the stationary wave is 4.0 cm .

What is the total distance travelled by a particle at an antinode during a time interval of 1.0 minute ?

- A 0 m
 B 72 m
 C 144 m
 D 192 m
☒ E 288 m
 F 576 m

$$\frac{\lambda}{2} = 4 \text{ cm} \rightarrow$$


$$\lambda = 8 \text{ cm}$$

$$v = f \lambda \therefore f = \frac{3.2}{0.08} = \underline{\underline{40 \text{ Hz}}}$$

$$\text{Total distance} = \underbrace{1.5 \text{ cm}}_{1 \text{ cycle}} \times \underbrace{4}_{2 \text{ waves}} \times \underbrace{2}_{1 \text{ cycle/s}} \times \underbrace{60}_{1 \text{ min}} \times 60$$

$$= 12 \times 2400 \text{ cm}$$

$$= 12 \times 24 \text{ m} = \underline{\underline{288 \text{ m}}}$$

- 6 A ray of light of single frequency f is travelling in a block of transparent material.
- The ray strikes the boundary between the block and air at an angle θ to the boundary.
- When $\theta = 65^\circ$ the ray is just at the threshold of being totally internally reflected.
- Which of the following is an expression for the wavelength of the light in the material?

(The speed of light in air is v_{air} .)

(A) $\frac{v_{\text{air}} \times \cos 65^\circ}{f}$

B $\frac{v_{\text{air}} \times \sin 65^\circ}{f}$

C $\frac{f}{v_{\text{air}} \times \cos 65^\circ}$

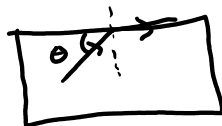
D $\frac{f}{v_{\text{air}} \times \sin 65^\circ}$

E $\frac{v_{\text{air}}}{f \times \cos 65^\circ}$

F $\frac{v_{\text{air}}}{f \times \sin 65^\circ}$

G $\frac{f \times \cos 65^\circ}{v_{\text{air}}}$

H $\frac{f \times \sin 65^\circ}{v_{\text{air}}}$



$$n_b \sin(90^\circ - \theta) = n_a \sin 90^\circ$$

$$n_b \cos \theta = n_a$$

$$n_b = \frac{c}{f \lambda_b} \quad n_a = \frac{c}{v_a}$$

$$\frac{n_b}{n_a} = \frac{1}{\cos \theta} = \frac{v_a}{f \lambda_b}$$

$$\therefore \lambda_{\text{block}} = \frac{v_a \cos 65^\circ}{f} //$$

- 7 A solid pyramid of height 140 m has a square base.

The density of the stone from which the pyramid is made is 2100 kg m^{-3} .

Atmospheric pressure is 100 kPa.

What is the average pressure on the ground under the pyramid?

(gravitational field strength = 10 N kg^{-1} ; volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{vertical height}$)

A 98 kPa

B 108 kPa

C 198 kPa

D 980 kPa

☒ E 1080 kPa

F 2940 kPa

G 3040 kPa

$$\text{Pressure} = \frac{\frac{1}{3} \times A \times 140 \times 2100 \times g}{A \times 1000} + 100$$

$$= 140 \times 7 + 100$$

$$= \underline{1080 \text{ kPa}}$$

- 8 The pressure exerted by a gas at constant temperature is directly proportional to its density.

A spherical bubble of gas forms at the bottom of a glass containing a fizzy drink.

The radius of the bubble at the point of formation, at the bottom of the drink, is R .

The depth of the liquid in the glass is h , and the density of the liquid of the drink is ρ .

Atmospheric pressure is P .

As the bubble rises, its radius changes.

Which expression gives the radius of the bubble when it is at a distance x below the surface of the drink?

(gravitational field strength = g ; volume of sphere = $\frac{4}{3}\pi r^3$ where r is the radius; the mass and the temperature of the gas in the bubble remain constant)

A $R \left(\frac{h\rho g - P}{x\rho g - P} \right)^{\frac{1}{3}}$

$$P \propto \rho \therefore P = k\rho$$

B $R \left(\frac{h}{x} \right)^{\frac{1}{3}}$

$$\text{Pressure} = P + \rho g x$$

C $R \left(\frac{h\rho g + P}{x\rho g + P} \right)^{\frac{1}{3}}$

$$pV = \text{constant}$$

D $R \left(\frac{x\rho g - P}{h\rho g - P} \right)^{\frac{1}{3}}$

$$\therefore (P + \rho g h) \left(\frac{4}{3} \pi R^3 \right) = (P + \rho g x) \left(\frac{4}{3} \pi r^3 \right)$$

$$(P + \rho g h) R^3 = (P + \rho g x) r^3$$

E $R \left(\frac{x}{h} \right)^{\frac{1}{3}}$

$$r = \left(\frac{P + \rho g h}{P + \rho g x} \right)^{\frac{1}{3}} R$$

F $R \left(\frac{x\rho g + P}{h\rho g + P} \right)^{\frac{1}{3}}$

- 9 A block of mass 2.0 kg slides directly down a smooth slope.

The slope is at an angle of 30° to the horizontal.

The block reaches a speed of 8.0 m s^{-1} , at which point the slope becomes rough and the block begins to decelerate.

After travelling a distance of 4.0 m down the rough slope the block comes to rest.

What is the magnitude of the average friction force between the block and the rough slope?

(gravitational field strength = 10 N kg^{-1} ; assume that air resistance is negligible)

A 2.0 N

B 6.0 N

C 10 N

D 12 N

E 16 N

F $10\sqrt{3}\text{ N}$

☒ G 26 N

H $(16 + 10\sqrt{3})\text{ N}$



$KE + GPE \rightarrow W$ by friction

$$\frac{1}{2}(2)(8^2) + 2(10)(4\sin 30) = F_f \times 4$$

$$64 + 40 = F_f \times 4$$

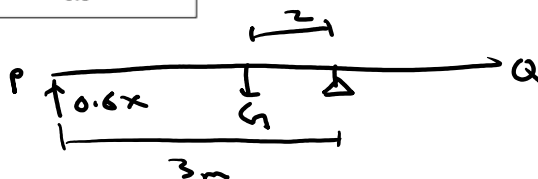
$$F_f = \frac{104}{4} = \underline{\underline{26\text{ N}}}$$

- 10 A non-uniform beam PQ of length 5.0 m and weight X rests on a pivot placed 3.0 m from end P. It is kept in equilibrium in a horizontal position by an upward force of magnitude $0.60X$ acting at end P.

The normal contact force at the pivot is 800 N.

What is the weight of the beam and how far is the centre of gravity of the beam from the pivot?

	weight of beam / N	distance from pivot / m
A	500	0.50
B	500	1.8
C	500	3.0
D	2000	0.50
E	2000	1.8
F	2000	3.0



$$R(\uparrow): 800 + 0.6X = X \Rightarrow X = \frac{800}{0.4} = 2000$$

$$\tau_Q: 0.6X(5) + 800(2) = 2000(2+2)$$

$$6000 + 1600 = 4000 + 2000 \cdot 2$$

$$2000 \cdot 2 = 3600 \Rightarrow 2 = \underline{\underline{1.8 \text{ m}}}$$

- 11 A car is travelling along a straight road with constant acceleration. It passes a road sign. It travels 12.2 m in the 3rd second and 14.4 m in the 4th second after passing the road sign. What was the speed of the car as it passed the road sign?

- A 2.20 ms⁻¹
 B 4.50 ms⁻¹
 C 6.70 ms⁻¹
 D 7.80 ms⁻¹
 E 13.3 ms⁻¹
 F 37.2 ms⁻¹

Speed at start of 2nd second = u

$$12.2 = ut + \frac{1}{2}at^2$$

$$12.2 = u + \frac{1}{2}a$$

At start of 3rd second, speed = v

$$14.4 = v + \frac{1}{2}a$$

$$a = \frac{v-u}{t} = \frac{v-u}{1} \therefore v = u + a$$

$$12.2 = u + \frac{1}{2}a$$

$$14.4 = u + \frac{3}{2}a \therefore a = 2.2 \text{ ms}^{-2}$$

$$u = 12.2 - 1.1 = 11.1 \text{ ms}^{-1}$$

$$v_{\text{sign}} = 11.1 - 2(2.2) = 6.7 \text{ m/s}$$

- 12 A light spring has unstretched length 0.40 m and spring constant 50 N m^{-1} .

The spring is stretched by a varying tension force that starts at a value of zero and increases at a constant rate of 0.20 N s^{-1} up to a maximum value.

When the force reaches its maximum value, the strain energy of the spring is 0.25 J.

What is the average power used to stretch the spring?

(Assume that the spring obeys Hooke's law.)

☒ A 0.010 W

B 0.020 W

C 0.040 W

D 0.080 W

E 1.0 W

F 2.0 W

G 4.0 W

H 8.0 W

$$\frac{1}{2} kx^2 = 0.25$$

$$kx^2 = 0.5$$

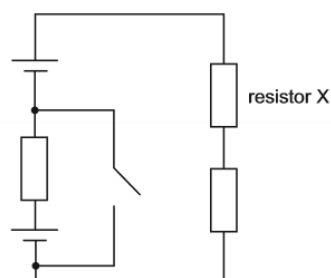
$$x^2 = \frac{0.5}{50} = 0.01$$

$$x = \underline{\underline{0.1 \text{ m}}}$$

$$F = kx = 50 \times 0.1 = 5 \text{ N}$$

$$\frac{5}{0.2} = 25 \text{ s} \therefore \text{Power} = \frac{0.25 \text{ J}}{25 \text{ s}} = \underline{\underline{0.01 \text{ W}}}$$

- 13 The circuit below contains three identical resistors, and two identical cells. When the switch is open, the power dissipated by resistor X is P .



What is the power dissipated by resistor X after the switch is closed?

- A $\frac{P}{4}$
 B $\frac{9P}{16}$
 C $\frac{3P}{4}$
 D P
 E $\frac{16P}{9}$
 F $\frac{9P}{4}$

$$\text{Open} \rightarrow V_{\text{tot}} = 2V$$

$$R_{\text{tot}} = 3R$$

$$I = \frac{2V}{3R}$$

$$P = \left(\frac{2V}{3R}\right)^2 \cdot R = \frac{4V^2}{9R} = P$$

$$\text{Closed} \rightarrow V_{\text{tot}} = V, R_{\text{tot}} = 2R$$

$$I = \frac{V}{2R}$$

$$P = \left(\frac{V}{2R}\right)^2 \cdot R = \frac{V^2}{4R}$$

$$\frac{V^2}{4R} \div \frac{4V^2}{9R} = \frac{9}{16}$$

- 14 A car of mass m is pulling a caravan of mass M .

The caravan is connected to the car by a metal bar of length l and cross-sectional area A .

The Young modulus of the metal from which the bar is made is E .

The car and caravan have a constant forward acceleration a and there are total resistive forces D_1 acting on the car and D_2 acting on the caravan.

What is the extension of the bar?

(Assume that the bar obeys Hooke's law and that the cross-sectional area of the bar remains unchanged.)

A $\frac{Mal}{EA}$

B $\frac{Ma}{EA}$

☒ C $\frac{(Ma + D_2)l}{EA}$

D $\frac{Ma + D_2}{EA}$

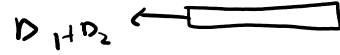
E $\frac{(Ma + ma + D_1 + D_2)l}{EA}$

F $\frac{Ma + ma + D_1 + D_2}{EA}$

$$\text{Caravan} \rightarrow T_{\text{bar}} - D_2 = Ma$$

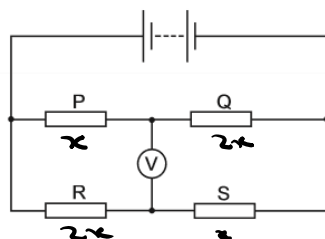
$$T_{\text{bar}} = Ma + D_2$$

$$x = \frac{FL}{AE} = \frac{(Ma + D_2)l}{EA}$$



- 15 Four resistors, P, Q, R and S, are connected to a battery with negligible internal resistance, as shown in the diagram.

P and S each have resistance x . Q and R each have resistance $2x$.



$$V_{\text{battery}} = V$$

Current \odot reading

$$= \frac{2V}{3} - \frac{V}{3} = \frac{V}{3}$$

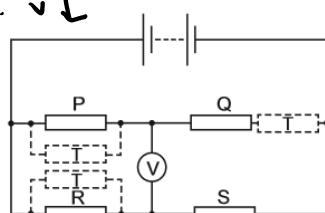
A fifth resistor, T, which has resistance x , is to be added to the circuit in one of the following listed positions, as shown in the diagram:

1 in parallel with P $\rightarrow R_P$ parallel \downarrow but R_S is the same

2 in series with Q $\rightarrow V_P \downarrow$ but R_S is the same $\therefore V \uparrow$

3 in parallel with R

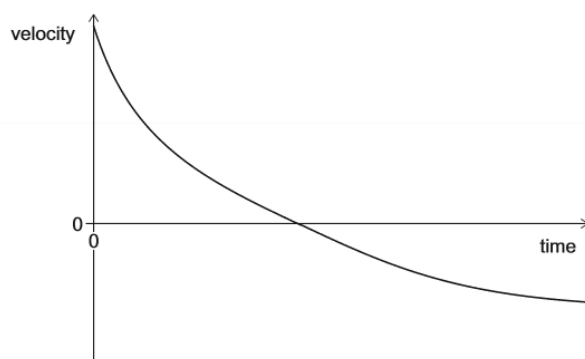
R_R parallel $\downarrow \therefore V \downarrow$



Which of the positions for resistor T causes an increase in the magnitude of the voltmeter reading?

- A none of them
- B 1 only
- C 2 only
- D 3 only
- E 1 and 2 only**
- F 1 and 3 only
- G 2 and 3 only
- H 1, 2 and 3

- 16 A ball is thrown vertically upwards in air. The ball travels upwards to reach its highest point and then falls back down to its initial starting position. The velocity–time graph for the ball is shown.



Which of the following statements is/are correct?

- ✓ The magnitude of the acceleration of the ball is only equal to the magnitude of the acceleration of free fall when it is at its highest point.
- ✗ The time taken for the upward journey of the ball is equal to the time taken for the journey back down to its starting position. *From Graph*
- ✓ The maximum increase in the gravitational potential energy of the ball is less than its initial kinetic energy and greater than its kinetic energy when it returns to its starting position.

At top, air resistance = 0 as $v=0$. $F=mg$

Air resistance causes dissipation of energy

- A none of them
- B 1 only
- C 2 only
- D 3 only
- E 1 and 2 only
- F 1 and 3 only**
- G 2 and 3 only
- H 1, 2 and 3

- 17 A stone is projected from level ground at an angle of 30° to the horizontal.

After 1.0s the stone lands on a ledge at height h above the level ground.

During this journey the vertical component of velocity of the stone is upwards for the first 0.60s and downwards for the remaining 0.40s.

What is the value of h ?

(gravitational field strength = 10 N kg^{-1} ; assume that air resistance is negligible)

- ☒ A 1.0m
- ☐ B 1.6m
- ☐ C 2.0m
- ☐ D 3.0m
- ☐ E 3.2m
- ☐ F 6.0m
- ☐ G 7.0m
- ☐ H 11m



$$\text{Vertical component of } u = u \sin 30^\circ \\ = \frac{u}{2}$$

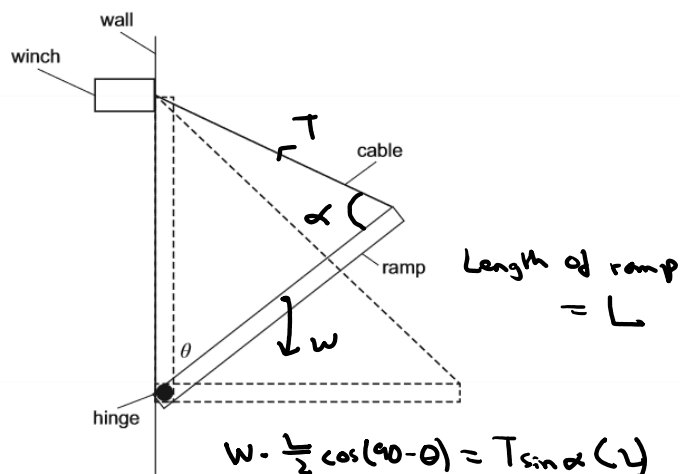
$$v = u + at$$

$$0 = \frac{u}{2} - 10(0.6) \text{ at top of parabola}$$

$$\frac{u}{2} = 6 \therefore u = 12 \text{ m/s}$$

$$s = ut + \frac{1}{2}at^2 = \frac{12}{2}(1) - 5(1^2) = \underline{\underline{1 \text{ m}}}$$

- 18 A drawbridge system consists of a uniform ramp, of weight W , that is smoothly hinged at its lower end. The upper end is connected by a light, inextensible cable to a winch that is fixed to the wall in the position shown in the diagram.



The ramp is lowered slowly, at constant speed, from its closed (vertical) position ($\theta = 0^\circ$) to its open (horizontal) position ($\theta = 90^\circ$).

What is the maximum tension in the cable during this process?

(double-angle identities: $\sin 2\theta = 2 \sin \theta \cos \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$)

A $\frac{W}{2}$

☒ B $\frac{W}{\sqrt{2}}$

C $\frac{\sqrt{3}W}{2}$

D W

E $\frac{2W}{\sqrt{3}}$

F $\sqrt{2}W$

G $2W$

$$\frac{W}{2} \sin \theta = T \sin \alpha$$

$$\alpha = \frac{180 - \theta}{2} = 90 - \frac{\theta}{2}$$

$$\frac{W}{2} \sin \theta = T \sin (90 - \frac{\theta}{2})$$

$$\sin (90 - \frac{\theta}{2}) = 1 \cos \frac{\theta}{2} + 0 \sin \frac{\theta}{2}$$

$$\frac{W}{2} (\cancel{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}) = T \cos \frac{\theta}{2}$$

$$T = W \sin \frac{\theta}{2} \quad | \quad T_{\max} \text{ when } \theta = 90^\circ$$

$$T = \frac{W \sqrt{2}}{2} = \frac{W}{\sqrt{2}}$$

- 19 A particle of mass m has kinetic energy E when it collides with a stationary particle of mass M . The two particles coalesce.

Which of the following expressions gives the total kinetic energy transferred to other forms of energy in the collision?

A 0

B $\frac{ME}{(M+m)}$

C $\frac{mE}{(M+m)}$

D $\frac{(M+m)E}{m}$

E $\frac{(M+m)E}{M}$

F $\frac{mME}{(M+m)^2}$

G E

$$E = \frac{1}{2}mv^2 \quad \therefore v = \sqrt{\frac{2E}{m}}$$

$$m\sqrt{\frac{2E}{m}} + M(0) = (m+M)u$$

$$u = \frac{m\sqrt{\frac{2E}{m}}}{m+M}$$

$$KE_{\text{after}} = \frac{1}{2}(m+M)(u^2)$$

$$= \frac{1}{2}(m+M) \left(\frac{\sqrt{2Em}}{(m+M)} \right)^2$$

$$= \frac{mE}{m+M}$$

$$E - \frac{mE}{m+M} = \frac{ME}{m+M} //$$

- 20 The critical angle for light incident on a boundary from medium X to air is 45° .

The critical angle for light of the same frequency incident on a boundary from medium Y to air is 60° .

There is a boundary between medium X and medium Y. Light of the same frequency travelling in one of these mediums is incident on this boundary.

In which direction of incidence is there a critical angle at this boundary, and within what range is this critical angle?

	direction of incidence	critical angle
A	X to Y	between 0° and 30°
B	X to Y	between 30° and 45°
C	X to Y	between 45° and 60°
D	X to Y	between 60° and 90°
E	Y to X	between 0° and 30°
F	Y to X	between 30° and 45°
G	Y to X	between 45° and 60°
H	Y to X	between 60° and 90°

X:

$$n_1 \sin 45 = n_{\text{air}} \sin 90 = 1$$

$$n_1 = \sqrt{2} \approx 1.41 \dots$$

Y:

$$n_2 \sin 60 = 1$$

$$n_2 = \frac{2}{\sqrt{3}} \approx 1.1$$

\therefore Incidence is X to Y

$$\sqrt{2} \sin \theta = \frac{2}{\sqrt{3}} \sin 1$$

END OF TEST

$$\sin \theta = \frac{2}{\sqrt{6}} > \frac{\sqrt{2}}{2} \text{ but } < \frac{\sqrt{3}}{2}$$